

國立高雄海洋科技大學 103 學年度碩博士班入學考試
海洋環境工程系-工程數學試題
【※不須使用計算機】

1. Solve the following ODE. (20%)

(a) $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$

(b) $xdx - ydy + y^3(x^2 - y^2)dy = 0$

2. Solve the following initial value problem. (20%)

$$y'' - 2y' + y = 2\sin 3x, \quad y(0) = 2, \quad y'(0) = 1$$

3. Find the general solution of the following Euler-Cauchy Equation. (20%)

$$x^4y^{(4)} + 6x^3y^{(3)} + 9x^2y'' + 3xy' + y = 0$$

4. Use the Power Series Method to find an approximative solution of following nonhomogeneous ODE and only demonstrate five terms. (20%)

$$y'' + 2y' = 4x^2y, \quad y(0) = 2, \quad y'(0) = 1$$

Hint: $y = \sum_{n=0}^{\infty} C_n x^n$

5. A definition of Laplace transform is $Y(s) = \mathcal{L}[y(t)] = \int_0^{\infty} e^{-st} y(t) dt$ and an inverse of Laplace transform is $y(t) = \mathcal{L}^{-1}[Y(s)]$. Solve the following initial value problem by Laplace transform. (20%)

$$y'' - 3y' + 2y = e^{3t}, \quad y(0) = 0, \quad y'(0) = 0$$

Hint: $\mathcal{L}[e^{at}] = \frac{1}{s-a}$, $\mathcal{L}[y'(t)] = sY(s) - y(0)$, $\mathcal{L}[y''(t)] = s^2 Y(s) - sy(0) - y'(0)$