1. Solve the following ODE. (20%)
   (a) \((2xy^4 e^x + 2xy^3 + y)dx + (x^2 y^4 e^x - x^3 y^2 - 3x)dy = 0\)
   (b) \(xdx - ydy + y^3(x^2 - y^2)dy = 0\)

2. Solve the following initial value problem. (20%)
   \(y'' - 2y' + y = 2\sin 3x, \ y(0) = 2, \ y'(0) = 1\)

3. Find the general solution of the following Euler-Cauchy Equation. (20%)
   \(x^4 y^{(4)} + 6x^3 y^{(3)} + 9x^2 y'' + 3xy' + y = 0\)

4. Use the Power Series Method to find an approximative solution of following nonhomogeneous ODE and only demonstrate five terms. (20%)
   \(y'' + 2y' = 4x^2 y, \ y(0) = 2, \ y'(0) = 1\)
   Hint: \(y = \sum_{n=0}^{\infty} C_n x^n\)

5. A definition of Laplace transform is \(Y(s) = \mathcal{L}_t[y(t)] = \int_0^\infty e^{-st} y(t)dt\) and an inverse of Laplace transform is \(y(t) = \mathcal{L}^{-1}[Y(s)]\). Solve the following initial value problem by Laplace transform. (20%)
   \(y'' - 3y' + 2y = e^{3t}, \ y(0) = 0, \ y'(0) = 0\)
   Hint: \(\mathcal{L}[e^{at}] = \frac{1}{s-a} \) , \(\mathcal{L}[y'(t)] = sY(s) - y(0), \ \mathcal{L}[y''(t)] = s^2Y(s) - sy(0) - y'(0)\)